

MusicMath[®]

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with

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4 Math & Music Lesson Ideas

(For 1st and 2nd grades)

1. Sorting and adding
2. Missing addends - musical cross-number puzzles
3. Systematic lists - How many ways to arrange items
4. Halving

Sorting and Adding



Part 1 - Musical Cross-Number Puzzles

Purpose:

The two main objectives in this lesson are to learn to sort objects with more than one attribute, and to use sorting as a vehicle for addition.

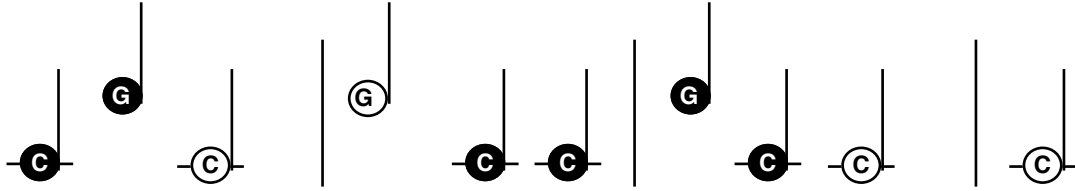
Introduce:

Remember how we sorted triangles and circles with different colors? Well we can do the very same thing with musical notes.

Notes too have shapes (demonstrates); We have Half Notes, that look like this , and have how many beats? [2], and we have Quarter Notes, that look like this , and have how many beats? [1]. But there is a difference. Notes don't have colors. They are always black. But they do have something that triangles and circles don't have. They have letter names, like C (plays a C on the keyboard) or G (plays a G). So when we sort notes we sort them by their beats, which shows up as their shape, and by their letter that tells us where to play them on the keyboard.

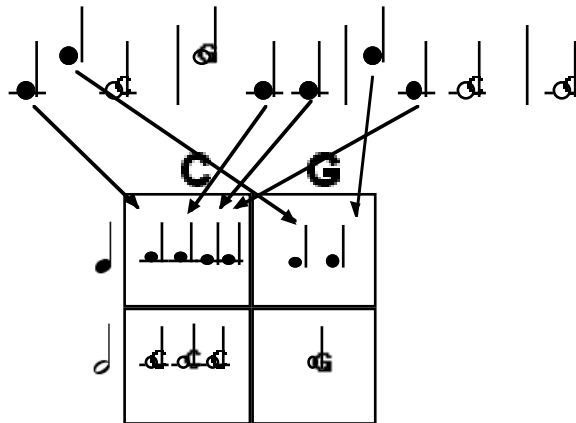
2. Teach:

Here is a melody. (Optional: Let's sing it together)

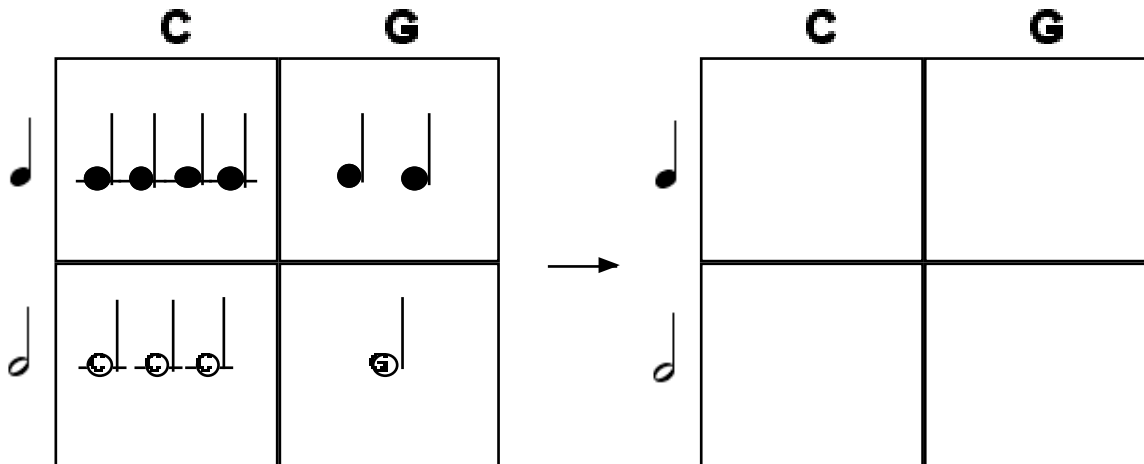


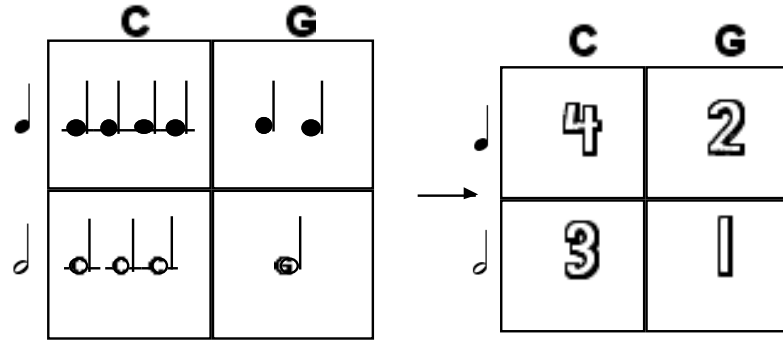
It has some Cs and some Gs, but notice that not all the Cs are the same, and not all the Gs are the same. Some Cs are Quarter Notes (point to them) and some are Half Notes, and the same is true for the Gs.

Can you help me sort these notes into their correct boxes:



Let's count how many notes are in each square and write those numbers in the empty squares on the right?



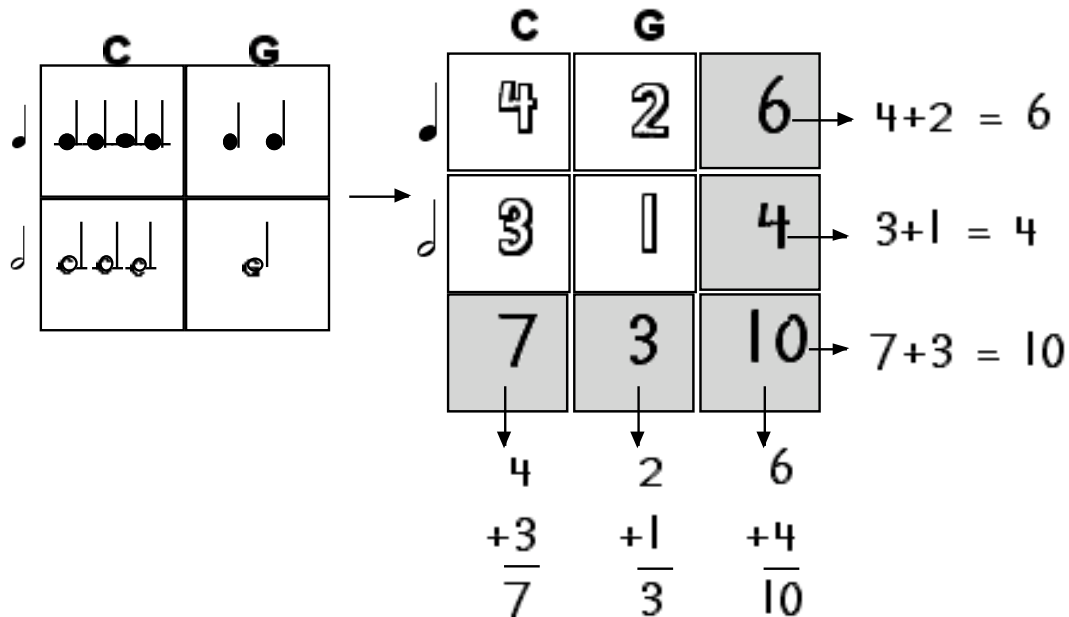


Can anybody tell me:

- How many Quarter Notes are there total (both Cs and Gs)? [6]
- And how many Cs are there total (both Quarter Notes and Half Notes)? [7]

There is an easy way to figure this up by adding up the squares sideways and down and writing the result in the shaded empty squares, like this:

[This is a Cross-Number-Puzzle applied to musical notation]:



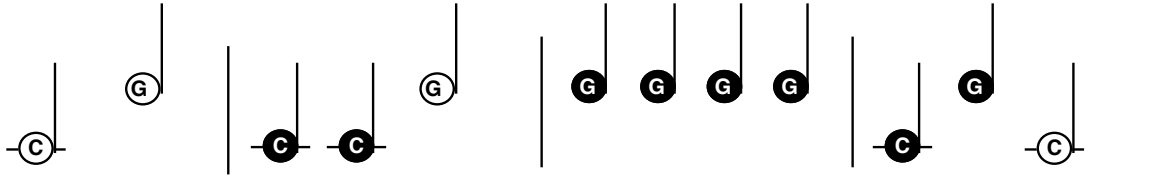
So now we can easily answer all these questions [point to each result square]:

- How many Half Notes are there? [Sideways: $3+1=4$]
- How many Gs are there? [Down: $2+1=3$]
- How many notes are there? [Right hand bottom corner: 10]
- How many Quarter Notes are there? [Sideways: $4+2=6$]
- How many Cs are there? [Down: $4+3=7$]


3. Students working individually or in pairs

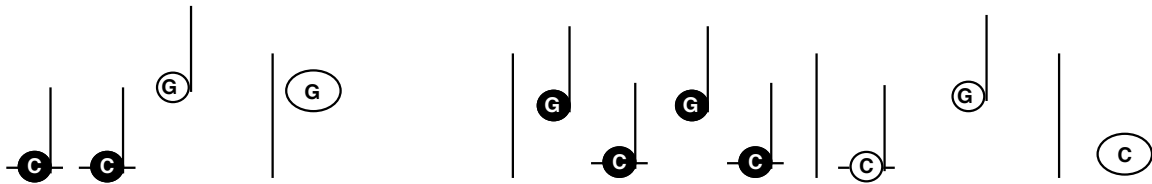
Worksheet No. 1 has two different melodies, one similar to the one we just did - Quarter Notes and Half Notes:

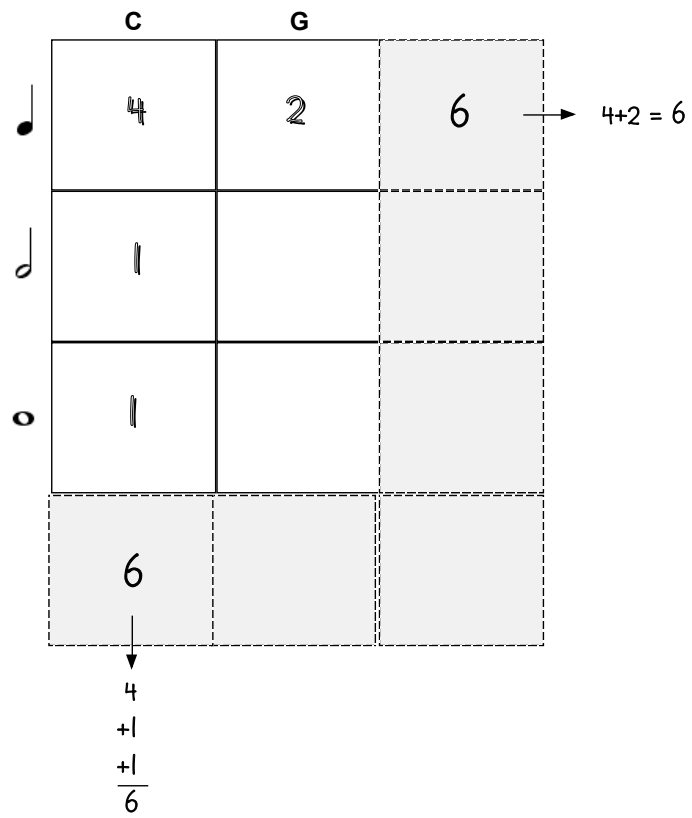
1.



2.

The second melody introduces a third attribute, a Whole Note , which requires a third row in our "spreadsheet":





4. Music

Let's play and count this last melody. Remember to count the extra held beats, saying "Hold" on the Half Notes, and "Hold, Hold, Hold" on the Whole Notes.

5. Discussion

Return to the ThinkMath Cross-Number Puzzles with some of the addends missing, as a preparation for the next lesson.

Sorting and Adding

Part 2 - Musical Missing Addends

1. Launch

Purpose:

This is a continuation of the previous lesson. In this lesson we use music to introduce missing addends. We use duration values of notes (Quarter Notes, Half Notes, etc.) as stand-ins for numbers.

Note to teacher:

You will notice that we purposely place the = sign and the missing addents in different positions, not just in the conventional: $a + \underline{\quad} = c$. This is common in music where addents (notes) can show up anywhere in a measure, and it is extremely useful for establishing the mental flexibility needed in arithmetic.

Introduce (review):

Remember the puzzle with notes and numbers you solved last week?

Here it is again:




	C	G	
♪	4	2	6
♩	1	2	3
♩	1	1	2
	6	5	11







We counted the notes in the melody and wrote the number in the right square, and then added up each *row* and each *column*.

2. Teach:

Today we will use the same kind of table to solve a different puzzle. Here is the same table except some of the numbers are missing. Can you help me fill in the blanks?




4		6
	2	3
1		2
6	5	11




I wonder what this same puzzle would look like if we replaced the numbers with musical notes? Remember that a  has 1 beat, a  has 2 beats, and a  has 4 beats:

		6 beats
		3 beats
		2 beats
6 beats	5 beats	11 beats

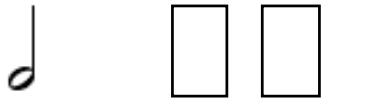
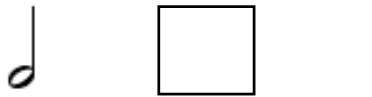
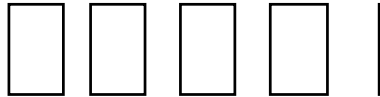
Now that we know how to replace missing numbers and missing notes, here are a few more puzzles for you to work out on your own.

3. Students working individually or in pairs

In this musical puzzle each row must add up to 4 beats but some of the notes are missing. Can you place a  or a  or a  in the empty squares?

(Remember that a  has 1 beat, a  has 2 beats, and a  has 4 beats)

4 Beats



And here is the same puzzle except it uses numbers instead of notes. On each little line fill in the missing numbers that will make the line add up to 4, like this:

$$\boxed{\text{♪}} \boxed{\text{♪}} \boxed{\text{♪}} \boxed{\text{♪}} \mid \rightarrow \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} = 4$$

$$\text{♪} \quad \boxed{\text{♪}} \mid \rightarrow 2 + \underline{\quad} = 4$$

$$\boxed{\text{♪}} \text{♪} \quad \boxed{\text{♪}} \mid \rightarrow \underline{\quad} + 2 + \underline{\quad} = 4$$

$$\text{♪} \quad \boxed{\text{♪}} \boxed{\text{♪}} \mid \rightarrow 2 + \underline{\quad} + \underline{\quad} = 4$$


$$\boxed{\text{♪}} \text{♪} \mid \rightarrow \underline{\quad} + 2 = 4$$

$$\boxed{\text{♪}} \boxed{\text{♪}} \text{♪} \mid \rightarrow \underline{\quad} + \underline{\quad} + 2 = 4$$

$$\boxed{\text{○}} \mid \rightarrow \underline{\quad} = 4$$

4. Teach:

Here is a different musical puzzle. This puzzle uses a new kind of note: A Dotted Half

Note. It looks like this , and it has 3 beats!

In this puzzle there are 2 measures with the same number of beats. The measure on the left has how many beats?

$$\text{♪.} \text{♪} \mid \boxed{?} \text{○} \mid \quad [5]$$

Can we find the missing note in the other measure and make it too add up to 5?



We can make a similar puzzle with numbers:

$$3 + 2 = \underline{\quad} + 5$$

[3+2=1+5]

And here is another puzzle:

$$3 + 4 = \underline{\quad} + 5$$

[3+4=2+5]

5. Students work on similar problems in a worksheet:

$$4 + \underline{\quad} = 5 + 2$$

$$3 + \underline{\quad} = 7$$

$$\underline{\quad} = 3 + 4$$

Systematic lists - How many ways to arrange items?

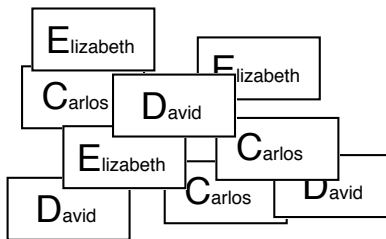
Launch

Purpose:

The focal point of this activity is figuring out how many solutions there are to a mathematical problem involving ways of arranging things.

Teach:

(Teacher asks students to pick 3 cards from a box. The cards have one of three names (different from before): either Carlos, Elizabeth, or David, and each name has the initial printed large:



Each student also gets a blank table to record different arrangements (only the initial):

(Note: Although there are only 6 possible arrangements, the table should have 10 rows, so as not to give away the answer).

When finished, students call out arrangements and the teacher records them on the board, eliminating duplicates.

“How can we tell that there are no more?”

One way to check it is to start with one child (e.g. Pedro) and see how many ways to arrange the rest (then start with a different child, etc.)

[Solution - these 6 arrangements listed in any order]:

Carlos David Elizabeth
Carlos Elizabeth David
David Elizabeth Carlos
David Carlos Elizabeth
Elizabeth David Carlos
Elizabeth Carlos David

2. Students working individually or in pairs

Worksheet problems

1. Let's do the same thing with music. We can make melodies with those first letters of the names: C-D-and E. Each melody will be just 3-notes long. How many different melodies can you make if each one has to use all 3 notes?

[Solution:

C-D-E
C-E-D
D-E-C
D-C-E
E-C-D
E-D-C]

2. Let's do the same thing with numbers. We'll make 3-digit numbers with the digits: 2, 4, and 5. How many different 3-digit numbers can you make if you use all 3 digits in each number?

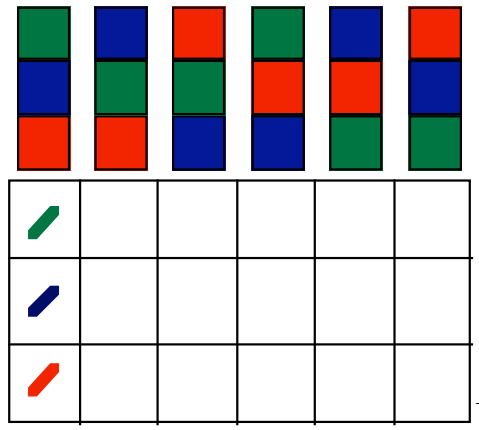
[Solution:

245
254
425
452
524
542]

3. [Manipulative using colored tiles (or blocks) and crayons]

Let's do the same thing with towers. Select 3 blocks with different colors, one of each. How many different towers can you make? Record your towers with your crayons.

[Solution: 6 towers



3. Discussion

“Let’s go back to problem 1 above where you made melodies. What melodies did you come up with?

Are there any others? Did we miss some? Do we have any duplicates?

How can we check that there are no more?”

[“One way to check it is to start with one letter and see in how many ways we can arrange the other two letters, then start with a different letter and keep doing the same.

“Other suggestions?”]

4. Music

Let’s play these melodies on the keyboard. Pick two 3-note melodies from the list, the first starting with C and the second ending in C. Play them in order on the keyboard.

Try that with a different pair of melodies.

5. Alternatives and extension

What about 4 notes? Start with the notes C, D, E, and the G below them. Find all the arrangements that end with G. Find all the arrangements that end with C. Play one arrangement that ends with G followed by another that ends with C.

[Note to teacher:

Using 4 elements instead of 3 would be a little harder, but in fact since one of the element doesn't change you are still only dealing with 3 letters just like before:

[Solution]

C	D	E	G
C	E	D	G
D	E	C	G
D	C	E	G
E	C	D	G
E	D	C	G

G	E	D	C
E	G	D	C
G	D	E	C
D	G	E	C
D	E	G	C
E	D	G	C

[Important:

The children don't need to find *all* the solutions to get the benefit of the activity!]

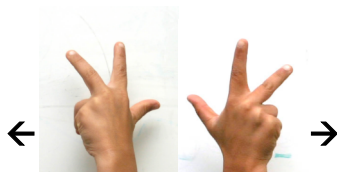
Halving

Purpose:

Learning distributive property and applying it to arithmetic, measuring, and music. The skills learned are useful for computations involving place value.

1. Launch

Half of 6?

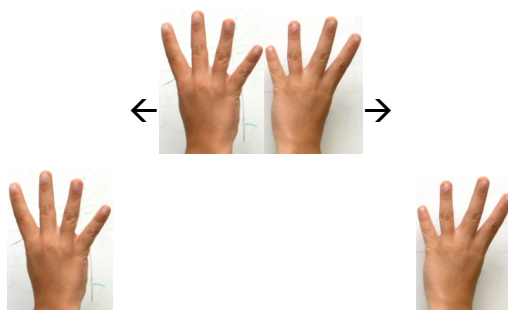


moving apart to become two groups of three fingers, like this:



No discussion or explanation, just a gesture that clarifies the communication and verifies that the answer is correct.

Half of 8?



“Too easy for you? Would you like another? (children generally chorus *yes*) Easier or harder? (they generally chorus *harder*) OK, how about half of (*pause*)...10?”

“What about 20?”




“What about 40?”






“Did you know that you can figure out half of forty-eight by taking half of 40 and half of 8? Who thinks they can figure out half of forty-eight?”

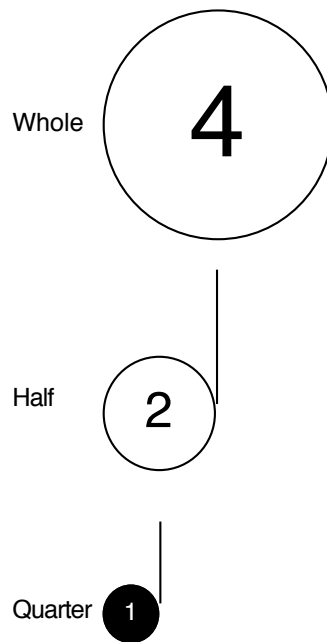
The task is hard *not* because of the arithmetic or because of the concept but because of the amount of focused attention and working memory involved. To do this, they have to keep 48, 40, 8, half of 40, and half of 8 in mind—remembering several numbers *and* remembering that they must take half more than once (but not more than twice!)—and then pick the right two of these numbers to recombine to get the answer. Many will get 24. A particularly common wrong answer is 28

2. Teach

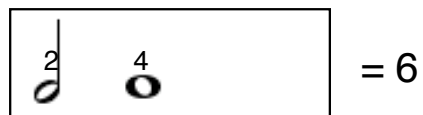
Now let's apply this same idea to musical notes.

Who can tell me what kind of note is this?  - A whole note. How many beats does a whole note have? **4**. What kind of note would have half of those 4 beats?  - A Half Note of course, which has **2** beats. What kind of note would be half of the half note?  - A Quarter note. And how many beats does a quarter note have (half of 2 is?), **1**. Here they are (have the children

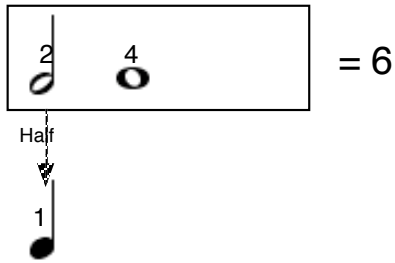
mime the shape of the notes with their hands and say the number of beats):     



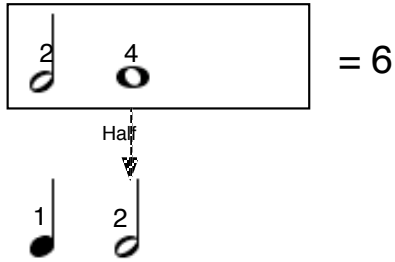
Here is a box containing a rhythm made up of a Half Note (which has 2 beats) and a Whole Note (which has 4 beats). How many beats does the box have? 6 beats (2+4=6).



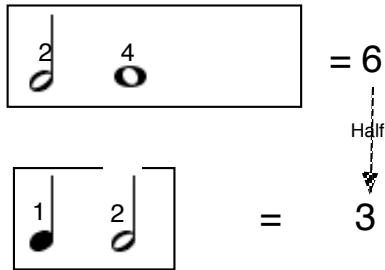
Suppose we cut each of the 2 notes in half, first we cut the Half Note in half, half of a Half Note is a - Quarter Note:



Then we cut the Whole Note in half and half a Whole Note is a - Half Note:

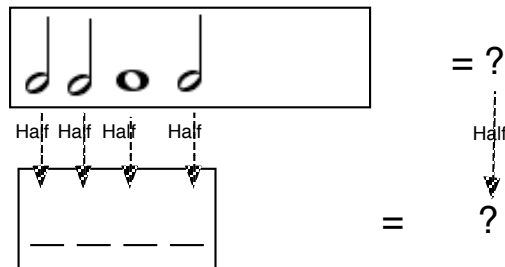


So, we have cut each note in half. Let's put them in a smaller box and count how many beats this smaller box has?



3, which is half of 6. Cool, huh? When we cut each note in half the total number of beats in the box got also cut in half!

[Preparation for independent work on worksheet. Project the total image.]



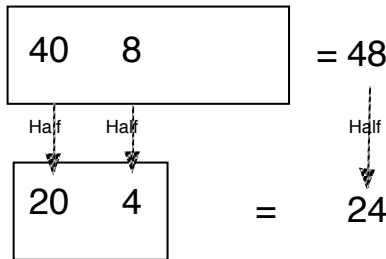
Let's try doing the same with a different rhythm pattern. Can you help me find:

The total of beats in the large box?
 Then cut each note in half?
 And how many beats are in the smaller box?
 Did it work?

Tie-in with Math

a. Numbers

This is the same thing we have done before with numbers. Remember how we cut 48 in half and found out that it was 24?



b. Measuring

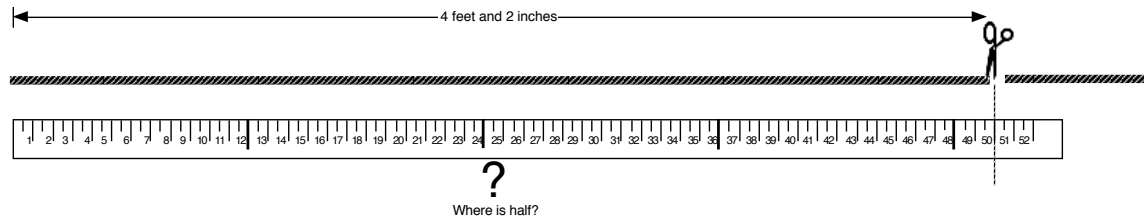
Let us use the same trick to cut in half a string that is 4 feet and 2 inches long. What do you think is half of 4 feet and 2 inches? (Half of 4 feet is 2 feet, and half of 2 inches is 1 inch). And the answer is?

[2 feet and 1 inch]

Let's try it and see if this works?

(Teacher distributes a 6 foot string and a tape measure to groups of 3 students.

Students measure out 4 feet and 2 inches and cut off the rest, then they find the half point of the string at 2 feet and 1 inch, and mark that point and fold the string to see if the two parts are the same length. (if not then one of the measurements is wrong and they have to do it again).



3. Students working individually or in pairs

Problem 1:

Let's go back to music. Here is a different rhythm pattern. Can you cut this rhythm in half the same way we did before, and see if the second box will have half the beats of the big box?

And now copy each of the new notes into the smaller box

[Teacher - Some students may not get all the notes right. This does not necessarily indicate that they did not comprehend the principle. Often students in a complex problem like this may forget to convert some of the notes, or in the effort to get it right forget what they were actually intending to accomplish. Checking the following constraints will help them fix any omissions or errors:

After solving the problem above ask students to check the following:

- Do I have the same number of notes as in the top box? (i.e. did I cut *all* the notes?)
- Did I cut each note in half? (or did I just copy a note without cutting it?)
- Count the beats in both boxes and check if the second box has half the beats of the first one?

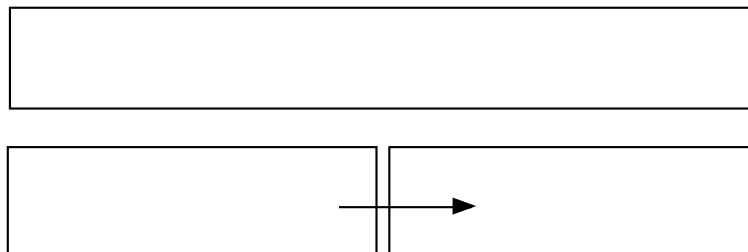
(Add other patterns on the worksheet)

4. Discussion - 2 halves make a whole

We figured out that half of 48 is 24. If two kids counted all the numbers at the same speed which would take longer to count, to 24 or to 48? [48 would take longer, in fact it would take twice as long, because 48 is twice 24]

The same thing is true for those boxes of beats. If you have a big box with 16 beats, and a small box with half the number of beats which is 8 beats, which would take longer to play the big box of 16 beats or the small box of 8 beats? [the 16 beats would take twice as long to play, because 8 is half of 16]. That could be a problem if two kids wanted to play the 2 rhythms at the same time.

The kid playing the small box would be finished in half the time it took the other kid to finish all the beats in the big box. So, the simplest thing is to have the kid playing the small box repeat it, like this:



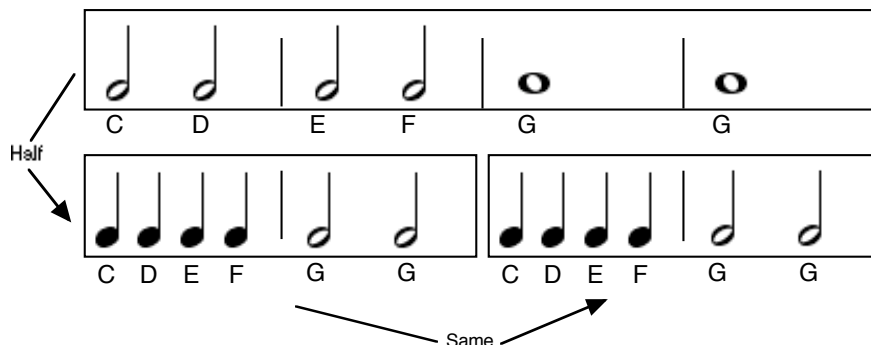
And guess what, since the smaller box is half of the big box if you just repeat the notes in it you'll finish exactly at the same time as the kid playing the notes in the bigger box.

Optional:

(If the music teacher is available she can divide the class in half and have each group clap one of the patterns. The teacher must prepare the kids to hold the longer notes for their full duration and not be swayed by the faster notes that are being clapped by the other group)

5. Music

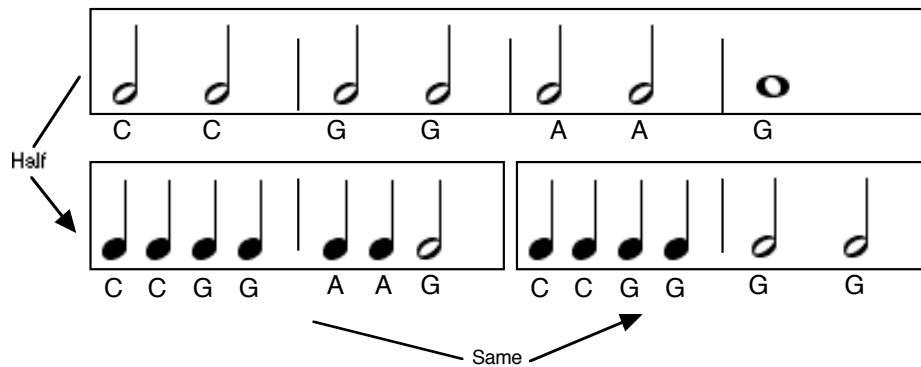
Let us take the pattern we worked on and assign notes letters to each of the note so we can make melodies out them:



Have teams of 3 kids play these two patterns at the same time. Two kids playing and the third clapping a steady beat.

Add other examples of melodies with other rhythms preferably recognizable tunes.

Example: Twinkle Twinkle Little Star:



6. Alternatives and extension

In the worksheet there should be a problem where the students create the pattern, with the following constraints:

- 6 notes
- Use any combination of Whole Notes and Half notes only